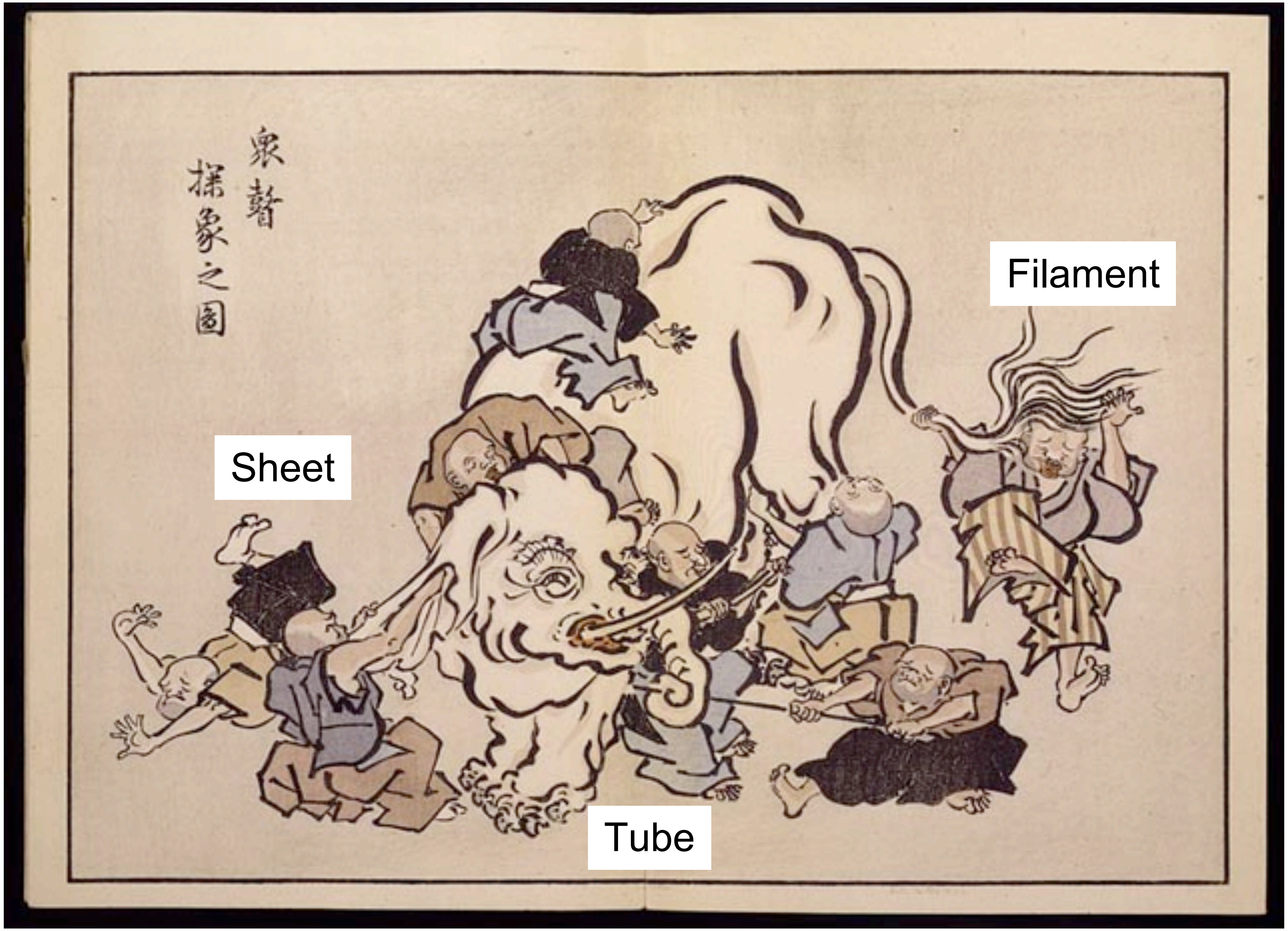
Conference on Small Scale Turbulence and Gradient Statistics
in memory of Kraichnan and Yaglom
Academia delle Scienze, Torino, October 26-29 2009

Round-Table

Akiva Yaglom
(1921-2008)

Robert Kraichnan
(1928-2008)

Buddhist parable of blind men examining an elephant



Hanabusa Itchô
(1652-1724)

J. Fluid Mech. (1974), *vol.* 62, *part* 2, *pp.* 305–330
Printed in Great Britain

On Kolmogorov's inertial-range theories

J. Fluid Mech,
62(2), 305-339
1974

By ROBERT H. KRAICHNAN
Dublin, New Hampshire

(Received 14 June 1973)

Consistency and uniqueness questions raised by both the 1941 and 1962 Kolmogorov inertial-range theories are examined. The 1941 theory, although unlikely from the viewpoint of vortex-stretching physics, is not ruled out just because the dissipation fluctuates; but self-consistency requires that dissipation fluctuations be confined to dissipation-range scales by a spacewise mixing mechanism. The basic idea of the 1962 theory is a self-similar cascade mechanism which produces systematically increasing intermittency with a decrease of scale size. This concept in itself requires neither the third Kolmogorov hypothesis (log-normality of locally averaged dissipation) nor the first hypothesis (universality of small-scale statistics as functions of scale-size ratios and locally averaged dissipation). It does not even imply that the inertial range exhibits power laws. A central role for dissipation seems arbitrary since conservation alone yields no simple relation between the local dissipation rate and the corresponding proper inertial-range quantity: the local rate of energy transfer. A model rate equation for the evolution of probability densities is used to illustrate that even scalar nonlinear cascade processes need not yield asymptotic log-normality. The approximate experimental support for Kolmogorov's hypothesis takes on added significance in view of the wide variety of *a priori* admissible alternatives.

If the Kolmogorov law $E(k) \propto k^{-\frac{5}{3}-\mu}$ is asymptotically valid, it is argued that the value of μ depends on the details of the nonlinear interaction embodied in the Navier–Stokes equation and cannot be deduced from overall symmetries, invariances and dimensionality. A dynamical equation is exhibited which has the same essential invariances, symmetries, dimensionality and equilibrium statistical ensembles as the Navier–Stokes equation but which has radically different inertial-range behaviour.

The stretching mechanism has led a number of authors to conjecture that the small-scale structure should consist typically of extensive thin sheets or ribbons of vorticity, drawn out by the stirring action of their own shear field (e.g., Townsend 1951; Batchelor 1953; Kraichnan 1959; Corrsin 1952; Saffman 1968). In this picture, the randomness lies in the distribution of thickness and extension of the thin sheets and ribbons, and in the way they are folded and tangled through the fluid. A typical small-scale structure is thought to be small in one or two dimensions only, not in the third.

Are there characteristic structures, length scales, …?

The 1941 theory is by no means logically disqualified merely because the dissipation rate fluctuates. On the contrary, we find that at the level of crude dimensional analysis and eddy-mitosis picture the 1941 theory is as sound a candidate as the 1962 theory. This does not imply that we espouse the 1941 theory. On the contrary, the theory is made implausible by the basic physics of vortex stretching. The point is that this question cannot be decided a priori; some kind of non-trivial use must be made of the Navier–Stokes equation.

Do we need more insight from Navier-Stokes, ...?

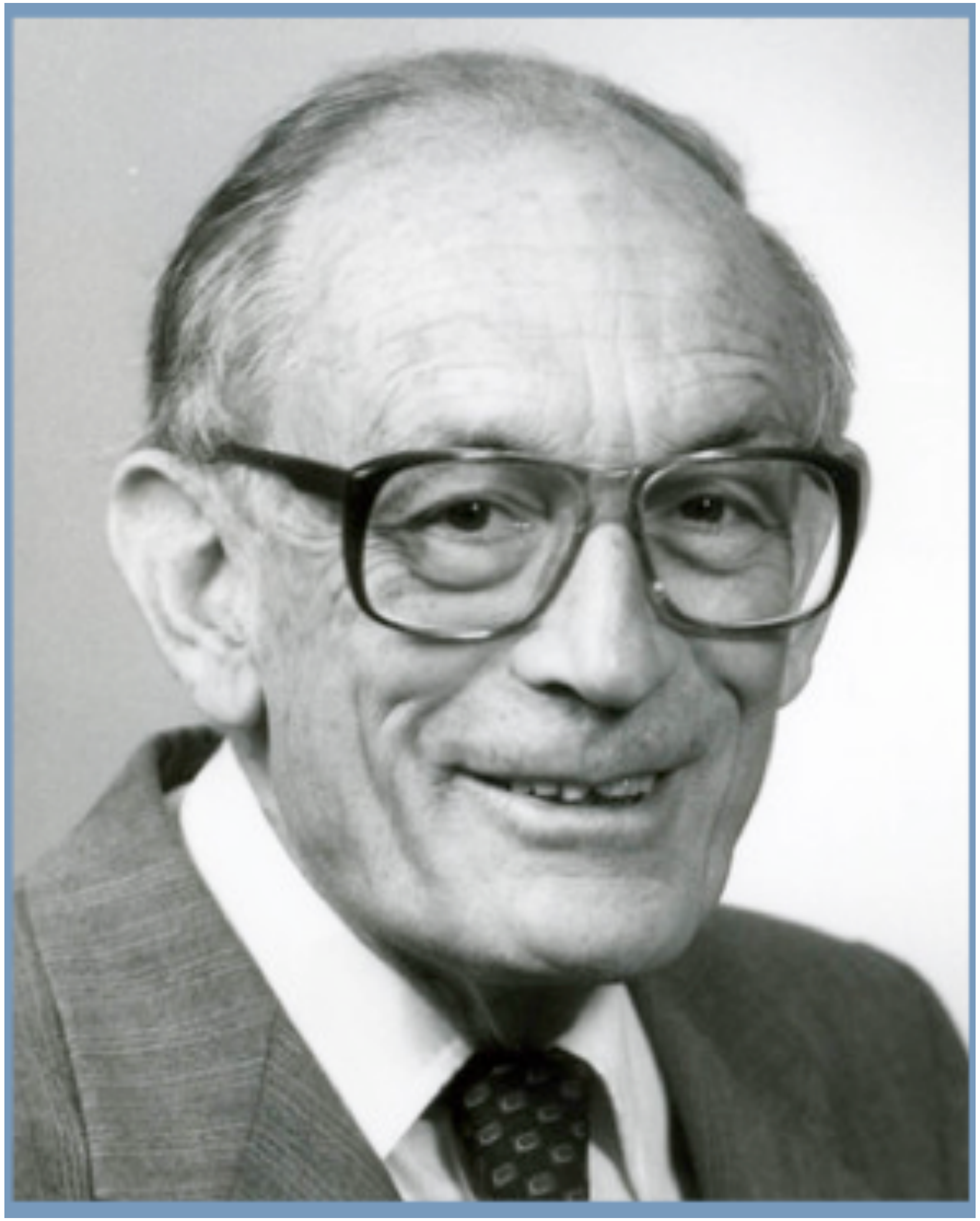
Katsushita Hokusai
(1760-1849)

The terms 'scale of motion' or 'eddy of size l appear repeatedly in the treatment of the inertial range. One gets an impression of little, randomly distributed whirls in the fluid, with the fission of the whirls into smaller ones, after the fashion of Richardson's poem. This picture seems to be drastically in conflict with what can be inferred about the qualitative structures of high Reynolds numbers turbulence from laboratory visualization techniques and from plausible application of the Kelvin's circulation theorem.

Do we have a better picture to propose, ...?

Our basic point is that the inertial-range cascade represents strong statistical disequilibrium. This carries two implications. First, that analogies with equilibrium and near-equilibrium phenomena are unjustified. Second, that the structure of the inertial range depends on the actual magnitude of the coefficients coupling the degrees of freedom and not just on their overall symmetry and invariance properties. This is because cascade is a transport process and the coefficient magnitudes affect the rate of transport.

Can we propose better hypotheses, …?



Hans Liepmann
(1914-2009)

It is clear that the essence of turbulent motion is vortex interactions. In the particular case of homogeneous isotropic turbulence this fact is largely masked, since the vorticity fluctuations appear as simple derivatives of the velocity fluctuations.

International Conference
on the Mechanics of Turbulence
September 2nd -9th 1961,
IMST, Marseille

Should we take the vorticity field more seriously, …?

As long as we are not able to predict the drag on a sphere or the pressure drop in a pipe from continuous, incompressible and Newtonian assumptions without any other complications, namely from first principles, we will not have made it!

Turbulence Workshop
UC Santa Barbara
1997

Should we bring the walls in, …?

The difficulty in defining the turbulent problem reminds me of a cartoon in which a rather dejected-looking researcher is introduced to a visitor: 'After twenty years of research, Dr. Quimsey developed the answer and now he has forgotten the question!' [… He] forgot the question, not because there was not one in the beginning, but because the path to a solution led into a maze of interconnected facts and problems, until his mind became as turbulent as the flow he was trying to describe.

The Rise and Fall
of Ideas in Turbulence,
American Scientist,
March-April1979

Katsushita Hokusai
(1760-1849)

Incidently,
what is the
question?