Order functions for statistical description of inhomogeneous turbulent flows

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In inhomogeneous turbulent flows, two processes, namely spatial transport and cascade in scales, take place simultaneously. Spatial inhomogeneity necessarily involves either large-scale coherent structures or thin boundary layers, and the accurate predictions of their spatial transport under a variety of flow conditions have always been a major challenge to statistical physics. A major source of complexity is a variety of flow structures in different flow regions and under different flow conditions. Mathematical models for turbulent flows should systematically enlarge the current scope of using eddy viscosity for modeling spatial transport and using ad hoc logarithmic mean velocity profile as the basic solution for wall-bounded flows. A concept of order function is proposed to describe non-equilibrium transport of energy and momentum, which can maximally take inputs from empirical data of experiments and/or numerical simulations. The order function is used to represent effects of turbulent structures, and hence may take a variety of structure-dependent forms. The methods of sub-ensemble average and long wavelength asymptotic analysis are proposed for studying relevant order functions, which then constitute a new form of closure equations for complex turbulent flows. It is speculated that the study of order functions may open the beginning of a theoretical framework, recently named as Structural Ensemble Dynamics (SED), which can describe the set of key quantities whose Reynolds number dependence have a clear pattern (empirical or theoretical), and which leads to an accurate prediction of mean-field properties (for engineers), and the order functions may constitute a basic part of fundamental study of turbulence, like energy spectrum for isotropic turbulence.

Briefly speaking, the SED approach addresses the statistical description of complex turbulent flow, which attempts to develop a link between the accurate description of the mean flow property, including mean velocity, mean shear and all relevant second-order mean quantities, and flow structural dynamics. Currently, the SED begins with collecting the empirical behavior of order functions, such as their Reynolds number dependence, Mach number dependence, geometry dependent behavior, etc. Careful analysis of these behaviors will lead to a closed theory for simple flows like channel and pipe flow, which is the simplest closed flow system extracted from the nature. Two immediate interests: 1) it provides a platform for analyzing the large DNS data, which is urgently needed; 2) it suggests a bridge between the fundamental research (about the effects of structures and hence the form of order-functions) and the practical engineering interests, since the accurate description of the mean property critically depend on the order-functions.