Velocity gradients statistics along particle trajectories in turbulent flows

Enrico Calzavarini[1]

[1] *Lab. de Physique, École Normale Supérieure de Lyon, CNRS UMR5672, 46 Allée d' Italie, 69007 Lyon, France.*

One of the most prominent features of turbulent flows are the strong variations in the energy dissipation field, a phenomenon called intermittency. In an attempt to describe quantitatively intermittent fluctuations in the inertial range of turbulence, Kolmogorov and Oboukhov in 1962 [1, 2] proposed a general relation linking velocity fluctuations, measured at a given spatial increment $\delta_r u = u(x+r,t) - u(x,t)$, with the statistical properties of the coarse grained energy dissipation, $\varepsilon_r = r^{-3} \int_{\Lambda(r)} \varepsilon(\boldsymbol{x})\, d^3x$ averaged over a volume, $\Lambda(r)$, of typical linear size r:

$$\delta_r u \sim r^{1/3} \varepsilon_r^{1/3}, \tag{1}$$

where $\sim$ means "scales as". Equation (1) is known as the Refined Kolmogorov Similarity Hypothesis (RKSH) and it is considered to be one of the most remarkable relations between turbulent velocity fluctuations: many efforts has been devoted in the last decades to its validation [3–5]. The RKSH relation bridges inertial-range properties with small-scale properties, supporting the existence of an energy cascade mechanism, statistically local in space. So far, a rather strong evidence supports the validity of the RKSH in the Eulerian frame (i.e. the laboratory frame). On the other hand, no investigation has been reported in the literature on the validity of RKSH in the Lagrangian frame. The main difficulty in studying RKSH in a moving reference frame stem from the necessity to make multi-point measurments along particle trajectories, in order to calculate the stress tensor. Also numerical experiments are very demanding, requiring refined computations of velocity differences along particle trajectories, something usually implemented by a heavy use of Fast Fourier Transform combined with off-grid interpolation. Here we report the first of such measurements using state-of-the-art Direct Numerical Simulations (DNS) with resolution up to 2048^3 collocation points, corresponding to $Re_\lambda \sim 400$. We present an investigation of the statistics of velocity gradients along lagrangian trajectories (i.e. fluid tracers trajectories) in a homogeneous and isotropic turbulent flow. The investigation is also extended to the trajectories of inertial particles: We show that the Lagrangian RKSH is well verified for time lags larger than the typical response time of the particle, τ_p. Implications of these findings for modeling of particle transport in many applied problems are also discussed.

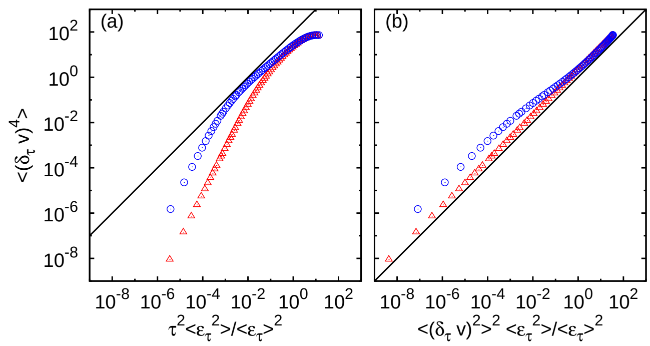

FIG. 1: Test of Lagrangian RKSH along the trajectories of fluid tracers and heavy particles at $\mathrm{Re}_\lambda = 400$. (a) For $p = 4$ we show the relation $\langle(\delta_\tau v)^p\rangle \simeq \tau^{p/2}\langle\varepsilon_\tau^{p/2}\rangle$ for fluid tracer (circles) and heavy particles with Stokes number $St = 2$ (triangles). Note that $\delta_\tau v$ and ε_τ denote respectively the velocity difference and the time coarse-grained energy dissipation measured along a particle trajectory (see [6]). (b) We show the validity of $\langle(\delta_\tau v)^p\rangle \simeq \left(\frac{\langle(\delta_\tau v)^2\rangle}{\langle\varepsilon_\tau\rangle}\right)^{p/2}\langle\varepsilon_\tau^{p/2}\rangle$ for the same values of p and St. Straight lines correspond to the theoretical scaling prediction.

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